

Ferromagnetic properties of charged vector boson condensate

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Abstract

Bose-Einstein condensation of W bosons in the early universe is studied. It is shown that, in the broken phase of the standard electroweak theory, the condensed W bosons form a ferromagnetic state with aligned spins. In this case the primeval plasma may be spontaneously magnetized inside macroscopically large domains and form magnetic fields which may be the seeds for the observed today galactic and intergalactic fields. However, in a modified theory, e.g. in a theory with stronger quartic self interactions of gauge bosons e.g. due to a smaller value of the weak mixing angle, antiferromagnetic condensation is possible. In the latter case W bosons form scalar condensate with macroscopically large electric charge density i.e. with a large average value of the bilinear product of W-vector fields but with microscopically small average value of the field itself.

1 Introduction

Bose-Einstein condensation is the quantum phenomenon of the accumulation of identical bosons in the same state, which is their lowest energy (zero momentum) state. Under these conditions they behave as a single macroscopic entity described by a coherent wave function rather than a collection of separate independent particles. Even though the Bose-Einstein condensation had been foreseen long time ago (1925), it took seventy years to make the first experimental observation, which was performed in a dilute gas of rubidium [1]. Difficulties in performing this observation were created by the extremal conditions necessary for the condensation. Indeed, the Bose-Einstein condensation takes place when the inter-particle separation is smaller than their de Broglie wavelength, $\lambda_{dB} \sim 2\pi/\sqrt{2mT}$, so the system must be cooled down to a very low temperature at ordinary densities.

In the recent years the study of Bose-Einstein condensate (BEC) became an active area of research in different fields of physics from plasma and statistical physics (for a review see book [2] and references therein) to astrophysics and cosmology.

The presence of charged Bose-Einstein condensates has interesting consequences in gauge field theories. For instance, in our recent papers [3, 4], electrodynamics of charged fermions and scalar bosons was considered with the fermion asymmetry above a certain critical value such that the boson chemical potential had to be equal to the boson mass. In these conditions the bosons should condense to ensure electric neutrality of the plasma. The screening of impurities in such plasma is essentially different from the case when the condensate is absent.

A similar problem has been recently considered also in the framework of an effective field theory [5, 6, 7]. The authors focused on the applications to the astrophysics of the helium white dwarfs discussing possible condensation of helium nuclei and analyzing the thermodynamical

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properties of the system and possible observational signatures. The formalism they used is complementary to ours, but our results agree in overlapping areas.

In this work we consider the BEC of charged W -bosons, which may be formed in the early universe, and study its magnetic properties. In general W bosons may condense both below and above the EW symmetry breaking if the cosmological lepton asymmetry happened to be sufficiently high, i.e. if the chemical potential of neutrinos was larger than the W boson mass at this temperature. The condensation of vector bosons differs from the theoretically simpler case of scalar bosons due to the presence of an additional degree of freedom, their spin states. In both cases, scalar and vector, the condensed bosons are in the zero momentum state but in the latter case the spins of the individual vector bosons can be either aligned or anti-aligned. These states are called respectively ferromagnetic and anti-ferromagnetic ones, see e.g. ref. [2]. The realization of one or the other state is determined by the spin-spin interaction between the bosons. In the lowest angular momentum state, $l = 0$, a pair of bosons may have either spin 0 or 2. Depending on the sign of the spin-spin coupling, one of those states would be energetically more favorable and would be realized at the condensation. In the case of the energetically favorable higher spin state, $S = 2$, the vector bosons condense with macroscopically large value of their vector wave function $\langle W_j \rangle$. In the opposite case of the favorable $S = 0$ state the vector bosons form the scalar condensate with pairs of vector bosons making a scalar “particle”. Such phenomena are observed in solid state physics with such spin-1 bosons as ^{23}Na , ^{39}K , and ^{87}Rb nuclei, see refs. [2], [8]–[11].

Usually experimental studies of the properties of the spin-1 condensate are performed in external magnetic fields. Under such conditions the spins of the vector bosons are aligned (frozen) due to the interactions of their magnetic moments with the external field. However, in optical traps an external magnetic field is absent and spin alignment or de-alignment depends upon the internal dynamics of the system. Correspondingly either ferromagnetic or scalar ground state would be formed depending on the scattering length of vector bosons in different angular momentum channels [8].

Let us stress again that in the ferromagnetic case the total spin of the vector bosons has macroscopically large value, while in the antiferromagnetic case the lowest energy state of many condensed vector bosons has zero total spin, $S_{tot} = 0$, if their number is even, and has $S_{tot} = 1$ (still macroscopically small), if their number is odd.

Due to macroscopically large value of the total spin in the ferromagnetic spin state the system can be accurately described by the mean field approximation, as is argued e.g. in sec. 12.2 of book [2] or refs. [10, 11]. Indeed, the validity of the mean field approximation is determined by the relative magnitude of the fluctuations near the ground state. The fluctuations are induced by the particle scattering which can change the spin value in a single reaction by ± 1 . It is clear that for a large value of the total spin the relative fluctuations $\delta S/S \sim \delta N/N \sim 1/\sqrt{N} \ll 1$, while for a small total spin value $\delta S/S \sim 1$.

In presence of sufficiently strong external magnetic field all the vector boson spins condense in the same direction and thus the whole body forms a single magnetic domain independently on the spin-spin interactions of the vector bosons, ferromagnetic or not. If however, an external magnetic field is absent, several magnetic domains would be formed in the ferromagnetic case, due to dynamical instability, and none in the antiferromagnetic case. The discussion of this phenomenon in solid state physics and the list of references can be found e.g. in ref. [12].

The situation with condensation of W -bosons is not as complicated as in solid state physics where the dominant contribution to the spin-spin interactions comes from the quantum exchange effects. However, for the system we are considering here, the exchange forces are not essential and the spin-spin interaction is determined by the interaction of the magnetic moments of the vector

bosons and by their self-interactions. The interactions with relativistic electrons and positrons are neglected, see below.

We show that charged vector bosons would condense in maximum spin states and form classical vector field, if only electromagnetic interactions between their spins are taken into account. In such a case the spontaneous magnetization at macroscopically large scales would take place. On the other hand, the local self-interaction of W creates the spin-spin coupling of the opposite sign. In the standard theory the magnitude of this coupling is smaller than that induced by the photon exchange, while the exchange of heavy Z -boson does not contribute at all into the spin-spin interactions of W -bosons. Thus the spin-spin coupling is dominated by the interactions of the magnetic moments of W . In pure electrodynamics magnetic fields are not screened and so one may expect that the plasma effects would not eliminate the dominance of the interactions between the magnetic moments. However, the situation is not clear in non-Abelian theories [13] and in principle the screening might inhibit the spin-spin magnetic interaction, see sec. 4.4. If this is realized, the local quartic selfcoupling of W would dominate over the electromagnetic one and W bosons should condense in antiferromagnetic state and form a scalar condensate. Hence a classical vector field would not be created.

The condensation of gauge bosons of weak interactions was considered in the pioneering papers [14], where it was argued that at sufficiently high leptonic chemical potential a classical W_j boson field could be created in the early universe. Our results are similar to that of ref. [14] as long as the ferromagnetic case is realized. In this case the spins of W add up coherently creating classically large average vector W_j boson field. In the hypothetical situation of a stronger quartic self-coupling of W we arrive to an opposite conclusion of vanishingly small classical W field but with macroscopically large number density of W -bosons at rest, which is given by the bilinear product $n_W = i(\partial_t W_j^\dagger W_j - W_j^\dagger \partial_t W_j)$ (as we see in what follows, this expression for the number density of W is true in the gauge where the electromagnetic potential is zero).

A different mechanism of formation of W -boson spin condensate by chaotic magnetic fields, which might exist in the early universe, was considered in ref. [15]. If such fields were sufficiently strong, this mechanism could operate independently on the spin-spin interaction of W -bosons and would align their spins in the domains with the size of the order of that of the original cosmic magnetic field, which are normally microscopically small. To this end a magnetic field with the strength $B > \alpha m_W^2$ would be necessary. Such fields might be generated at the electro-weak phase transition. The alignment of the W spins reminds the alignment of vector fields in magnetic traps mentioned above. Moreover, such an alignment under the influence of a sufficiently strong external magnetic field would take place in both ferromagnetic and antiferromagnetic cases. However when the external magnetic field is switched-off or redshifted, the spins of the “antiferromagnetic” W -bosons would be dis-aligned making scalar condensate, while in the ferromagnetic case macroscopically large domains with aligned spins would be created. The mechanism of formation of such domains is different from the normal ferromagnets due to an absence of the exchange forces. So probably the size of the domains is not determined by the usual competition between the volume and surface energies but by the causality effects.

A recent application of vector BEC to astrophysics was studied in ref. [16], where the condensation of deuterium nuclei was considered. The authors argue that the interaction between deuterium nuclei forces them into the lowest spin antiferromagnetic state.

The paper is organized as following. In sec. 2 kinetic equation approach to BEC is considered. In sec. 3 the same phenomenon is studied on the basis of the equations of motion of the vector fields. In sec. 4 the spin-spin interaction of W bosons in the zero momentum state is calculated. In sec. 5 discussion of the results and conclusion are presented.

2 Formation of Bose-Einstein condensate. Kinetic approach

It is convenient to describe formation of BEC using kinetic theory approach. Let us consider a system of bosons and fermions in thermal equilibrium with temperature T . As is known, the equilibrium distribution functions, up to the spin counting factor, are equal to:

$$f_{F,B} = \frac{1}{\exp[(E - \mu_{F,B})/T] \pm 1}, \quad (1)$$

where the signs plus and minus stand respectively for fermions and bosons and $\mu_{F,B}$ are chemical potentials of fermions and bosons. Evidently the chemical potential of bosons cannot exceed their mass, $\mu_B \leq m_B$. Otherwise their distribution would not be positive definite. This upper bound on μ_B enforces the Bose-Einstein condensation if the asymmetry between bosons and anti-bosons is so high that even maximally large chemical potential, $\mu_B = m_B$, is not sufficient to provide for such a high asymmetry. In this case the bosonic distribution function takes the form:

$$f_B = C\delta^{(3)}(\mathbf{p}) + \frac{1}{\exp[(E - m_B)/T] - 1}, \quad (2)$$

where the first delta-function term describes the condensate and a new constant parameter C is its amplitude. The second term, which is the usual Bose distribution, describes non-condensed particles and vanishes at $T = 0$.

It is easy to verify that the distributions written above annihilate the collision integral, i.e. they are the equilibrium distributions. Indeed the kinetic equation has the form:

$$\frac{df_1}{dt} = I_{coll}[f], \quad (3)$$

where the collision integral is equal to

$$I_{coll} = -\frac{(2\pi)^4}{2E_1} \int d\tau'_{in} d\tau_{fin} \delta^{(4)} \left(\sum p_{in} - \sum p_{fin} \right) |A_{if}|^2 F[f], \quad (4)$$

and

$$F[f] = \Pi f_{in}(1 \pm f_{in}) - \Pi f_{fin}(1 \pm f_{in}). \quad (5)$$

In eq. (4) $d\tau'_{in}$ is the phase space of all initial particles except for particle under scrutiny, i.e. the initial particle 1, $d\tau_{fin}$ is the phase space of all final particles, A_{if} is the amplitude of the transition from an initial to a final state. The product is taken over all initial (in) and final (fin) states and signs plus or minus stand for bosons and fermions respectively. It is straightforward to check that $F[f]$ vanishes on distributions (1) for arbitrary μ_F and $\mu_B \leq m_B$ satisfying the usual condition of chemical equilibrium:

$$\sum \mu_{in} = \sum \mu_{fin}. \quad (6)$$

If $\mu_B = m_B$, there arises an additional freedom that $F[f]$ vanishes for distribution (2) with an arbitrary C . The value of the latter is determined by the magnitude of the asymmetry between bosons and anti-bosons. Notice that in equilibrium chemical potentials of particles and antiparticles are opposite, $\bar{\mu} = -\mu$. Hence if bosons condense with $\mu_B = m_B$, the anti-bosons cannot condense because $\mu_{\bar{B}} = -m_B$.

We assume above that the collision amplitude is T-invariant but even if this restriction is lifted the collision integral is still annihilated by functions (1) or (2) due to S-matrix unitarity [17].

As is stated in the Introduction, a large lepton asymmetry is necessary to make W bosons condense. In particular, the condensate is formed at lepton number densities higher than the critical one $n_\nu^c = m_W^3/6\pi^2$. When $T \rightarrow 0$, the W -boson mass, m_W , approaches its usual vacuum value, created by the Higgs condensate. When the temperature rises, there are two opposite effects on m_W . The first one is the usual positive temperature correction $\delta m_W \sim eT$. The second effect is negative and is related to a decrease of the amplitude of the Higgs condensate. As a result, at temperatures above the electroweak phase transition, when the Higgs condensate disappears [18], the sole contribution to W -mass comes from the temperature corrections [19] and m_W may be much smaller than its vacuum value.

The temperature dependence of n_ν^c below the EW phase transition was analyzed in refs. [20, 21]. It should be noted here that, if T is higher than the critical temperature of the EW phase transition, the W condensation may take place also for lower, but still large, values of the lepton asymmetry [21]. In this case, as is mentioned above, the W mass would be essentially determined by the medium effects, i.e. by the temperature corrections or by the condensate itself. Thus the W mass in the electroweak symmetric phase might be lower than in the broken phase and a lower lepton asymmetry would be required for the condensation. In this paper we mostly assume that the temperature is below the electroweak phase transition and thus the plasma consists of massive W and Z bosons, neutral Higgs bosons, quarks, leptons, and their antiparticles. Nevertheless the interesting possibility of W condensation at lower lepton asymmetry should be kept in mind.

Models of generation of large lepton asymmetry were considered in refs. [22]-[25]. However, if the asymmetry was generated above the electroweak (EW) phase transition, it might be transformed into the baryonic one by the sphaleron processes, creating unacceptably large baryon asymmetry. So we need to assume that the lepton asymmetry should be created below the EW phase transition.

On the other hand, a mechanism to avoid generation of too large baryon asymmetry could be triggered by a large lepton asymmetry itself. It was pointed out in ref. [14] and confirmed in several subsequent papers [26, 23] that a large lepton asymmetry suppresses restoration of the electroweak symmetry and hence if lepton asymmetry was generated at very high temperatures when sphalerons were not active and the electroweak symmetry was broken at these high temperatures, the electroweak baryon non-conservation would never be efficient.

Finally, concerning the formation of the Bose-Einstein condensate, it is essential that the particles in question possess a conserved quantum number that forces their chemical potential to be nonvanishing, if the number of particles is not equal to the number of antiparticles. The amplitude C is then calculated from the known difference of the number densities of particles and antiparticles.

In this paper we consider a simple example of electrically neutral plasma in which charged W -bosons condense because of a large asymmetry between leptons and antileptons. For simplicity we confine ourselves to only one family of leptons. This simplification does not influence the essential features of the result. A more detailed analysis with all the particles included can be found in ref. [27]. Quarks may be essential for the imposing of the condition of vanishing of all gauge charge densities in plasma and for the related cancellation of the axial anomaly but we work in the lowest order of the perturbation theory where the anomaly is absent.

The plasma is supposed to be electrically neutral, with zero baryonic number density but with a high leptonic one. The essential reactions are the direct and inverse decays of W :

$$W^+ \leftrightarrow e^+ + \nu. \quad (7)$$

The equilibrium with respect to these processes imposes the equality between the chemical po-

tentials:

$$\mu_W = \mu_\nu - \mu_e \quad (8)$$

The condition of electroneutrality reads:

$$n_{W+} - n_{W-} - n_{e-} + n_{e+} = 0 \quad (9)$$

The leptonic number density is equal to

$$n_L = n_\nu - n_{\bar{\nu}} + n_{e-} - n_{e+} . \quad (10)$$

Here $n = g_s \int d^3p / (2\pi)^3 f$ is the number density of the corresponding particles and g_s is the spin counting factor. One should remember that only left-handed electrons participate in reaction (7), chirality is conserved in reactions with Z -bosons and photons, and chirality flip may take place only in reactions with Higgs bosons.

Leptonic number is supposed to be conserved, so n_L is constant in the comoving volume and is a fixed parameter of the scenario. (B+L)-nonconservation induced by sphalerons is neglected because temperature is mostly assumed to be below the electroweak phase transition.

It is evident that for sufficiently high n_L the chemical potential of W should reach its maximum value, $\mu_W = m_W$, and with further rising n_L , W -bosons must condense.

Let us first assume that the density of leptonic charge is very large, $n_L > T^3$. Correspondingly the amplitude of the condensate must also be large, $n_W^C \approx C / (2\pi)^3 > T^3$. In this case $\mu_{e,\nu} > m_W$ and equations (8–10) can be easily solved: $\mu_\nu \approx \mu_e$, $n_L \approx \mu_\nu^3 / 2\pi^2$, and $n_W^C \approx (2/3)n_L$. Interparticle separation of W -bosons under these conditions is $d \sim n_L^{-1/3} < T^{-1}$, while the de Broglie wave length of the condensed W 's with zero momentum is formally infinitely large. In the realistic condensate the particle momentum is not precisely zero but is of the order of the inverse size, d^{-1} , of the region where W -bosons condense. Another relevant quantity is the de Broglie wave length of W -bosons above the condensate, $\lambda_{dB} \sim 2\pi / \sqrt{2mT}$. The condition $\lambda_{dB} > d$ would be fulfilled as long as $T < m_W / 2\pi^2$.

A huge cosmological lepton asymmetry, $n_L \gg T^3$, could be created in the Affleck-Dine scenario [28]. In this model the universe could be quite cold. Later when $n_L \sim 1/a^3$ is diluted by the cosmological expansion down to the value when μ_W becomes smaller than m_W the condensate would evaporate and the universe would be reheated.

Though the possibility of a huge lepton asymmetry is quite interesting, the condensation of W -bosons could take place even with much smaller $n_L \sim m_W^3$, as can be seen from the analysis of the kinetics presented in this section. The interparticle separation of W -bosons under these conditions is $d \sim n_L^{-1/3} \sim m_W^{-1}$. In this case the temperature may be relatively high, $T \simeq m_W$, but nevertheless W -bosons could condense. According to the equilibrium distribution (2) the plasma would consist of two parts, the condensate with zero momentum and the high temperature plasma over the condensate. The de Broglie wavelength of the high temperature plasma would be again $\lambda_{dB} \sim 2\pi / \sqrt{2mT}$, that is larger than d as long as $T < 2\pi^2 m_W$. As is argued above, the de Broglie wave length of the condensed W -bosons even in this case is much larger than the interparticle distance.

Thus at $T \sim m_W$ the charged weak bosons, W , condense if all the relevant quantities are close by an order of magnitude to the W -boson mass to a proper power, i.e. $\mu_\nu \sim \mu_e \sim m_W$, $n_L \sim m_W^3$. They rise with rising temperature, as can be found from numerical solution of eqs. (8–10).

We will also discuss the W condensation above the EW phase transition. In this situation the Higgs condensate is absent and the W -bosons acquire, due to temperature corrections, $m_W \sim gT$ [19], where $g \sim 0.1$ is the typical coupling constant.

The critical values of chemical potential necessary to achieve W condensation at lower n_L have been calculated, even though an analytic solution is possible only in some special limits and the approximations used are not valid in some regions of parameters. The general equations together with some critical discussion concerning the approximations used can be found in the literature, see [20, 21]. For instance, at high temperature, that is T much larger than masses and chemical potentials but smaller than the EW breaking value, W -bosons would condense if $\mu_L > 0.3\sqrt{T^2 - T_c^2}$, where μ_L is the chemical potential associated to the total lepton number and $T_c \sim 200$ GeV [21]. Another limit that can be analytically calculated is the low temperature one ($T \sim 0$). Under this hypothesis, W -bosons condense when $\mu_L > 88.4 \text{ GeV} + T^2/64.8 \text{ GeV}$.

A caveat for a large lepton asymmetry arises from the consideration of the big bang nucleosynthesis with strongly mixed neutrinos. It is shown [29] that leptonic chemical potentials of all neutrino flavors are restricted by $|\mu_\nu/T| < 0.07$ at the BBN epoch. However, it should be noted that the entropy release from the electroweak epoch down to the BBN epoch diminishes the lepton asymmetry, n_L/n_γ , by the ratio of the particle species present in the cosmological plasma at these two epochs, which is approximately 10. Hence the original lepton asymmetry, $(n_L/n_\gamma)_{EW}$ could be of order unity and still compatible with BBN. It should be also noted that the BBN bound can be avoided [30] if neutrinos are coupled to the hypothetical pseudogoldstone boson, Majoron.

Moreover, at high temperatures, when the Higgs condensate is underdeveloped, the W -boson mass may be noticeably smaller than the plasma temperature and W could condense even at $|\mu/T| < 0.07$. If W s condensed above the electroweak phase transition, the magnitude of the chemical potential necessary for the condensation was much smaller than temperature, $\mu_W \sim gT$, as explained above. As we see, in this case the chemical potentials of electrons and neutrinos would be also much smaller than temperature. In this limit the differences between number densities of fermions and antifermions are:

$$n_F - n_{\bar{F}} = \frac{1}{6} g_F \mu_F T^2, \quad (11)$$

where g_F is the number of the spin degrees of freedom, $g_e = 2$, $g_\nu = 1$.

Substituting this expression into eqs. (8–10) and assuming an arbitrary chemical potential $\mu_W < T$, we find that the condensate would be formed if approximately $n_L \geq gT^3$ and that all the chemical potentials are of the order of gT . Correspondingly the necessary lepton asymmetry could be as small as $n_L/n_\gamma \sim g$. We see that even without the entropy release such lepton asymmetry is not dangerous for BBN.

3 Equations of motion of gauge bosons and their condensation.

In this section we consider the evolution of the gauge bosons of $SU(2) \times U(1)$ electroweak model and analyze the possibility of the condensate formation as supplementary to the approach of the previous section. In addition to that, using the equations of motion allows to take into account the spin-spin interactions of the vector bosons and to check if the condensate is ferromagnetic or antiferromagnetic.

The Lagrangian of the minimal electroweak model has the well known form:

$$L = L_{gb} + L_{sp} + L_{sc} + L_{Yuk}, \quad (12)$$

which are respectively the gauge boson, the spinor, the scalar, and the Yukawa contributions. Explicitly we have:

$$L_{gb} = -\frac{1}{4}G_{\mu\nu}^i G^{i\mu\nu} - \frac{1}{4}f_{\mu\nu}f^{\mu\nu}, \quad (13)$$

$$G_{\mu\nu}^i = \partial_\mu A_\nu^i - \partial_\nu A_\mu^i + g\epsilon^{ijk}A_\mu^j A_\nu^k, \quad f_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad i = 1, 2, 3$$

$$L_{sp} = \bar{\Psi} i \not{D} \Psi, \quad D_\mu \Psi = \left(\partial_\mu - \frac{i}{2}g \sigma^j A_\mu^j - \frac{i}{2}g' Y B_\mu \right) \Psi,$$

$$L_{sc} = \frac{1}{2}(D_\mu \Phi)^\dagger (D_\mu \Phi) + \frac{1}{2}\mu^2 \Phi^\dagger \Phi - \frac{1}{4}\lambda(\Phi^\dagger \Phi)^2, \quad D_\mu \Phi = \left(\partial_\mu - \frac{i}{2}g \sigma^j A_\mu^j - \frac{i}{2}g' B_\mu \right) \Phi,$$

and the Yukawa Lagrangian describes interactions of fermions with the Higgs field.

In the expressions above A_μ^i and B_μ are the gauge boson potentials of the $SU(2)$ and $U(1)$ groups respectively, g and g' are their gauge coupling constants, Y is the hypercharge operator corresponding to the $U(1)$ group and σ^j are the Pauli matrices operating in $SU(2)$ space. As usually, the repeated indices imply summation.

In the broken phase the physical massive gauge boson fields are obtained through the Weinberg rotation:

$$W_\mu^\pm = \frac{A_\mu^1 \mp iA_\mu^2}{\sqrt{2}}, \quad Z_\mu = c_W A_\mu^3 - s_W B_\mu, \quad A_\mu = s_W A_\mu^3 + c_W B_\mu, \quad (14)$$

where c_W and s_W stand respectively for $\cos \theta_W$ and $\sin \theta_W$ and θ_W is the Weinberg angle. The other fields involved in the Lagrangians presented above are the spinor Ψ and the Higgs field $\Phi = [\phi^+, \phi^0]^T$. The latter, after the symmetry breaking, acquires non-zero vacuum expectation value, v , and takes the form $\Phi = (1/\sqrt{2})[0, v + \phi_1^0]^T$, where the upper index T means transposition and ϕ_1^0 describes excitations in the broken symmetry phase, i.e. neutral massive Higgs particle. We are considering here one lepton family but the results can be easily generalized to the three family model. We use the unitary gauge in which the particle content is explicit, for example physical gauge bosons have three polarization states and only one physical neutral Higgs field, ϕ_1 is present.

From the presented above equations one can conclude that in addition to the usual kinetic and mass terms, vector boson interactions contain the following cubic and quartic couplings, see e.g. ref. [31]:

$$L_3 = ie \cot \theta_W \left[(\partial^\mu W^\nu - \partial^\nu W^\mu) W_\mu^\dagger Z_\nu - (\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}) W_\mu Z_\nu + W_\mu W_\nu^\dagger (\partial^\mu Z^\nu - \partial^\nu Z^\mu) \right] \\ + ie \left[(\partial^\mu W^\nu - \partial^\nu W^\mu) W_\mu^\dagger A_\nu - (\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}) W_\mu A_\nu + W_\mu W_\nu^\dagger (\partial^\mu A^\nu - \partial^\nu A^\mu) \right], \quad (15)$$

$$L_4 = -\frac{e^2}{2\sin^2 \theta_W} \left[(W_\mu^\dagger W^\mu)^2 - W_\mu^\dagger W^{\mu\dagger} W_\nu W^\nu \right] - e^2 \cot^2 \theta_W \left[W_\mu^\dagger W^\mu Z_\nu Z^\nu - W_\mu^\dagger Z^\mu W_\nu Z^\nu \right] \\ - e^2 \cot \theta_W \left[2W_\mu^\dagger W^\mu Z_\nu A^\nu - W_\mu^\dagger Z^\mu W_\nu A^\nu - W_\mu^\dagger A^\mu W_\nu Z^\nu \right] \\ - e^2 \left[W_\mu^\dagger W^\mu A_\nu A^\nu - W_\mu^\dagger A^\mu W_\nu A^\nu \right]. \quad (16)$$

Using the standard Euler-Lagrange procedure we can obtain the following Maxwell equations for the electromagnetic field:

$$\begin{aligned}\partial_\mu F^\mu{}_\nu &= J_\nu^\psi + ie \left[W_\mu^\dagger \partial_\nu W^\mu - W^\mu \partial_\nu W_\mu^\dagger + W_\nu^\dagger \partial_\mu W^\mu - W_\nu \partial_\mu W^\mu{}^\dagger + 2W^\mu \partial_\mu W_\nu^\dagger - 2W^\mu{}^\dagger \partial_\mu W_\nu \right] \\ &+ e^2 \left[2A_\nu W_\mu^\dagger W^\mu - A^\mu (W_\mu^\dagger W_\nu + W_\mu W_\nu^\dagger) \right] \\ &+ e^2 \cot \theta_W \left[2Z_\nu W_\mu^\dagger W^\mu - Z^\mu (W_\mu^\dagger W_\nu + W_\mu W_\nu^\dagger) \right],\end{aligned}\quad (17)$$

where A_μ is the electromagnetic potential, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the Maxwell tensor, and J_ν^ψ is the electromagnetic current of the charged fermions. All the rest in the r.h.s. of eq. (17) can be understood as the electromagnetic current of W bosons, J_μ^W . In what follows we assume that the total electric charge density of the plasma is zero and the average three-vector current vanishes as well.

Equations of motion for the other vector fields, W^\pm and Z , can be obtained from the Lagrangians (15, 16) plus the contributions from the kinetic and mass terms:

$$\begin{aligned}\partial_\mu W_\nu^\mu + m_W^2 W_\nu &= ie [A^\mu W_{\mu\nu} - \partial_\mu (W^\mu A_\nu) + \partial_\mu (W_\nu A^\mu) - W^\mu F_{\mu\nu}] \\ &+ ie \cot \theta_W [Z^\mu W_{\mu\nu} + \partial_\mu (Z^\mu W_\nu) - \partial_\mu (W^\mu Z_\nu) + W^\nu Z_{\nu\mu}] \\ &+ (e^2/\sin^2 \theta_W) [W_\nu (W_\mu^\dagger W_\mu) - W_\nu^\dagger (W^\mu W_\mu)] + e^2 \cot^2 \theta_W (W_\nu Z_\mu Z^\mu - Z_\nu Z_\mu W^\mu) \\ &+ e^2 \cot \theta_W (2W_\nu Z_\mu A^\mu - Z_\nu W_\mu A^\mu - A_\nu W_\mu Z_\mu) + e^2 (W_\nu A^\mu A_\mu - A_\nu W_\mu A_\mu), \quad (18) \\ \partial_\mu Z_\nu^\mu + m_Z^2 Z_\nu &= ie \cot \theta_W [W^\mu W_{\mu\nu}^\dagger - W^\mu{}^\dagger W_{\mu\nu} + \partial_\mu (W^\mu W_\nu^\dagger) - \partial_\mu (W^\mu{}^\dagger W_\nu)] \\ &+ e^2 \cot^2 \theta_W (2Z_\nu W_\mu^\dagger W^\mu - W_\nu^\dagger W_\mu W^\mu - W_\nu W_\mu^\dagger Z^\mu) \\ &+ e^2 \cot \theta_W (2A_\nu W_\mu^\dagger W^\mu - W_\nu^\dagger W_\mu A^\mu - W_\nu W_\mu^\dagger A_\mu),\end{aligned}\quad (19)$$

where $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, $V_\mu = W_\mu, Z_\mu$ and we assumed that the fermionic currents coupled to W and Z are absent. To avoid confusion it is worth noting that the field strength defined after eq. (13) includes the gauge boson self-coupling, while it is not included into $V_{\mu\nu}$.

Equations (18,19) are the equations for the corresponding field operators. They describe classical fields in the tree approximation and do not include the effects of W and Z instability. The latter can be taken into account by perturbative iteration of these equations. The effects of instability are discussed below.

We will consider the case when the electric charge density of fermions, J_0^ψ , is non-vanishing and homogeneous. It is usually assumed that the primeval plasma is electrically neutral and thus the non-zero charge density of fermions must be compensated by the opposite sign charge density of W . To study such a system it is convenient to use the electromagnetic gauge freedom and to impose the condition $A_\mu = 0$. We also assume that the average value of $Z_\mu = 0$. In this case there exists a homogeneous solution of the equation of motion of the form:

$$W_\mu = C_\mu \exp(-im_W t), \quad (20)$$

where we impose the condition $\partial_\mu W^\mu = 0$ to eliminate the non-physical spin degrees of freedom. So C_μ is a constant vector with vanishing time component and m_W is the boson mass determined by the nonzero vacuum expectation value of the Higgs field⁴. In contrast, in some papers

⁴We want to stress an important difference between Bose-Einstein condensate and Higgs condensate, which is often overlooked. The equation of state of the former is simply $P = 0$, while for the latter $P = -\rho$, where P and ρ are respectively pressure and energy densities.

the gauge condition of time independent charged vector field is taken: $W_\mu = C_\mu$, while the electromagnetic vector potential is non-zero: $A_\mu = \delta_\mu^t m_W / e$.

In addition to the Higgs induced part, the mass of W_μ contains also the contributions induced by the temperature effects [19] and by the impact of the W -condensate itself, which are disregarded at this stage, see discussion in sec. 2 about possible condensation of W at lower lepton asymmetry.

Solution (20) corresponds to the Bose condensate of W -bosons describing a collection of these particles at rest. This field configuration corresponds to the condensation of the positively charged W^+ . The condensation of W^- is described by the complex conjugate expression. However, such a solution is not obligatory and depends upon the kinetics of the system and the interactions between W -bosons at rest. In fact the only condition which we have at this level is the condition of the electric neutrality and it demands only that the average values of bilinear combinations of W must be non-zero, while classical vector field W_μ may vanish on the average. A possible vanishing of the classical vector field W_j , where $j = 1, 2, 3$ is the spatial vector index, is physically quite evident. The condensate is a collection of W -bosons at rest with the charge density which compensates the charge density of fermions. The most probable state of such particles (the highest entropy state), if the spin-spin interaction is neglected, is the state with chaotic distribution of the individual spins. It is natural to expect that the average value of the total spin in such a state is zero or at least not macroscopically large. The situation is opposite in the ferromagnetic case when the spin alignment is energetically favorable and the classical vector field could be formed.

In reference [14] a similar statement of the formation of a classical vector field was done but without an analysis of the dynamics introduced by the spin-spin interaction. In the quoted paper the mentioned above gauge of time independent W is used, but the physical results are of course gauge invariant. In this gauge the condition of the charge neutrality becomes $2A_0 W_\mu^+ W_\mu^- = -J_0^\psi$, which is again bilinear in W field and from the condition of non-zero charge density of the condensed W 's does not follow that there exists the classical field $W_j \neq 0$.

One more comment may be in order here. The instability of W can be introduced in the usual way by adding an imaginary part to the mass equal to the decay half-width. The introduction of such a term into the equation of motion for W leads to the exponential decay of the field, $W \sim \exp(-\Gamma t/2)$. However this description is valid only for the decay into vacuum.

For the decay into a dense medium the Fermi exclusion principle should be taken into account. Hence, if neutrinos have sufficiently large chemical potential, such that all the states where W could decay would be occupied, the decay rate would be exponentially suppressed and solution (20) could be physically realized. This observation establishes the equivalence between the kinetic approach of sec. 2 and that presented here.

In fact the absolute stability of the condensed W 's is unnecessary. Even if W -bosons decay, the ferromagnetic condensate can be formed, if the time of the condensate formation is shorter than the life-time of W -bosons in the plasma. The condensed W -bosons are in the state of dynamical equilibrium: W 's evaporating from the condensate because of their decay or scattering of hot fermions, are compensated by W 's coming back by the inverse processes. In thermodynamical equilibrium the average number of W -bosons in the zero momentum state remains constant. The decay rate of the W -bosons in plasma is proportional to

$$\Gamma \sim \frac{\Gamma_0}{[e^{-(m_W/2 + \mu_e)/T} + 1][e^{-(m_W/2 - \mu_\nu)/T} + 1]}, \quad (21)$$

where $\Gamma_0 \sim \alpha m_W$ is the decay rate of W in vacuum. The denominator in eq. (21) comes from the Fermi suppression terms $(1 - f_{e^+})(1 - f_\nu)$. We consider the decay $W^+ \rightarrow \nu e^+$ and take into account that in thermal equilibrium the chemical potentials of electrons and positrons are

equal by magnitude but have the opposite signs. In the case of very large lepton asymmetry, $n_L, \mu_\nu \gg T$, the decay rate of W -bosons would be exponentially suppressed and their life time can be longer than the time of the spin alignment, τ_S . As we mentioned above, the cosmological generation of $L \gg T^3$ may be realized in a version of the Affleck-Dine scenario [28] which leads to a cold universe with non-relativistic W -bosons. Moreover, even in absence of the exponential suppression and relatively small lepton asymmetry, $L \sim T^3$, the life-time of W -bosons in the plasma can be larger than τ_S . As is shown below, the Hamiltonian of spin-spin interaction is given by eqs. (29,34). Correspondingly the characteristic time of the spin alignment can be estimated as:

$$\tau_S \sim 1/U^{spin} \sim m_W^2/(n_W e^2 S^2), \quad (22)$$

where S is the total spin of the condensed W -bosons, $n_W \equiv 1/d^3$ is their number density, and d is the average distance between them. We approximated $\delta(\mathbf{r})$ as $1/d^3 = n_W$. Evidently τ_S can be considerably shorter than the life-time of W -bosons. The same “stability” arguments are applicable to the decay of W into quarks.

4 Spin-spin interactions of W bosons

As we mentioned above, the form of the vector boson condensate depends upon the interaction between the vector bosons at rest. If the latter favors the opposite spin configuration, i.e. a pair of bosons “prefers” to be in the zero spin state, the condensate would have zero total spin, i.e. W -bosons would form the scalar condensate (antiferromagnetic case). In the opposite situation of the favorable spin-two state, the spins of all vector bosons in the condensate would be aligned and the condensate would have macroscopically large spin (ferromagnetic case).

First, in subsection 4.1 we will consider only the spin-spin interaction induced by the electromagnetic interactions of W -bosons, namely by the coupling of their magnetic moments. Next, in subsection 4.2 we will take into account the local quartic self-coupling of W . In subsection 4.3 we take into account the spin-spin interaction caused by the Z -boson exchange between W 's. In sec. 4.4 we discuss the effects of possible screening of the magnetic moment interactions by plasma effects. In section 4.5 we discuss the results of the previous subsections and conclude.

4.1 Electromagnetic interactions

The essential particles in the system we study in this section are the charged W bosons at rest and charged relativistic fermions, which ensure the electric neutrality of the medium. The latter are electrons (or positrons) and quarks but these details are not important. For relativistic fermions helicity is conserved and hence the interaction of their spins with the spins of W is not essential, because on the average the electron-positron medium is not spin-polarized. Accordingly we take into account only the spin-spin interaction of non-relativistic W -bosons and disregard the impact of the charged fermions. The electromagnetic interaction between W -bosons is similar to the well known interaction of nonrelativistic electrons, which is described by the Breit equation. The derivation of this equation can be found e.g. in book [32]. According to the Breit equation the interaction between the magnetic moments of two electrons (i.e. their spin-spin interaction) has the form:

$$U^M(r) = \frac{e^2}{16\pi m_e^2} \left[\frac{(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)}{r^3} - \frac{3(\boldsymbol{\sigma}_1 \cdot \mathbf{r})(\boldsymbol{\sigma}_2 \cdot \mathbf{r})}{r^5} - \frac{8\pi}{3}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\delta^{(3)}(\mathbf{r}) \right], \quad (23)$$

where $\sigma_{1,2}$ are the spin operators of the electrons, i.e. the Pauli matrices averaged respectively over the first and second electron wave functions, and $e^2 = 4\pi\alpha = 4\pi/137$ (to avoid confusion let us note that in ref. [32] the notation is different, namely $e^2 = \alpha$).

The expression for potential (23) created by the one photon exchange is true for virtual photons propagating in vacuum. The presence of plasma changes the propagator and could modify the spin-spin potential. This is discussed in section 4.4. The screening effects are not important, at least in some temperature range.

The analogue of the Breit equation for W -bosons can be derived along exactly the same lines as is done for electrons. The electromagnetic interaction between two W bosons in the lowest order in the electric charge, e , is described by the usual one-photon exchange diagram. In the Feynman gauge, where the photon propagator is $D^{\mu\nu} = g^{\mu\nu}/q^2$, the amplitude corresponding to this diagram is:

$$M = -\frac{1}{(p_1 - p_2)^2} W^{\alpha'\dagger} W^{\beta'\dagger} V_{\alpha'\alpha\mu}(p_1, p_2, q) V_{\beta'\beta}{}^\mu(p_3, p_4, -q) W^\alpha W^\beta \quad (24)$$

where $V_{\alpha\beta\mu}$ is the most general CP invariant $W^\dagger W \gamma$ vertex [33]:

$$V_{\alpha\beta\mu}(p_1, p_2, q) = ie \left[p_\mu g_{\alpha\beta} + 2(q_\beta g_{\alpha\mu} - q_\alpha g_{\beta\mu}) + (1 - k_\gamma)(q_\beta g_{\alpha\mu} - q_\alpha g_{\beta\mu}) + \left(\frac{\lambda_\gamma}{2m_W^2} \right) p_\mu q_\alpha q_\beta \right], \quad (25)$$

where p_1 and p_3 are the momenta of the incoming particles, p_2 and p_4 are the momenta of the outgoing particles and $p = p_1 + p_2$, $q = p_2 - p_1$. This expression should be symmetrized with respect to the interchange of W -bosons in the initial and/or in the final states.

The vertex written above contains two anomalous coupling parameters k_γ and λ_γ . As we can see from eq. (17), the standard electroweak model predicts, up to the second order in the electromagnetic coupling constant e : $k_\gamma = 1$, $\lambda_\gamma = 0$. In what follows we mostly assume that these values are true, since they are compatible with the present experimental data for triple gauge boson couplings [34]. In this case the amplitude (24) reduces to:

$$M = \frac{e^2}{q^2} W_{1'}^{\alpha'\dagger} W_{2'}^{\beta'\dagger} [p_\mu g_{\alpha\alpha'} + 2(q_\alpha g_{\alpha'\mu} - q_{\alpha'} g_{\alpha\mu})] [p^\mu g_{\beta\beta'} - 2(q_\beta g_{\beta'}{}^\mu - q_{\beta'} g_\beta{}^\mu)] W_1^\alpha W_2^\beta. \quad (26)$$

The spin-spin interaction is contained in the product of the last two terms in the square brackets in eq. (26) i.e. the terms containing vector q . The spin operator of vector particles is defined as the generator of the rotation group belonging to its adjoint representation and is equal to the vector product:

$$\mathbf{S}_1 = -i \mathbf{W}_{1'}^\dagger \times \mathbf{W}_1 \quad (27)$$

Hence the scattering amplitude induced by the interaction between the magnetic moments of the charged vector bosons is equal to:

$$M_S = -\frac{e^2 \rho^2}{m_W^2 q^2} [q^2 (\mathbf{S}_1 \cdot \mathbf{S}_2) - (\mathbf{S}_1 \cdot \mathbf{q})(\mathbf{S}_2 \cdot \mathbf{q})] \quad (28)$$

where ρ is the ratio of the real magnetic moment of W to its value predicted by the standard electroweak theory (e^2/m_W^2) and we divided by $4m_W^2$ for proper normalization of the W -wave function, as is explained below, see Sec. 4.2.

The potential which describes the electromagnetic spin-spin interaction is the Fourier transform of amplitude (28) and is equal to:

$$U_{em}^{spin}(r) = \frac{e^2 \rho^2}{4\pi m_W^2} \left[\frac{(\mathbf{S}_1 \cdot \mathbf{S}_2)}{r^3} - 3 \frac{(\mathbf{S}_1 \cdot \mathbf{r})(\mathbf{S}_2 \cdot \mathbf{r})}{r^5} - \frac{8\pi}{3} (\mathbf{S}_1 \cdot \mathbf{S}_2) \delta^{(3)}(\mathbf{r}) \right]. \quad (29)$$

This potential has the same form as the corresponding one in the Breit's equation for electrons (23) but with the different numerical coefficient.

To calculate the contribution of this potential into the energy of two W -bosons at rest we have to average it over their wave function. In particular, in the condensate case, it is a S-wave function that is angle independent. Hence the contributions of the first two terms in eq. (29) mutually cancel out and only the third one remains, which has negative coefficient. Thus the energy shift induced by the spin-spin interaction is equal to:

$$\delta E = \int \frac{d^3 r}{V} U_{em}^{spin}(r) = -\frac{2e^2 \rho^2}{3Vm_W^2} (\mathbf{S}_1 \cdot \mathbf{S}_2), \quad (30)$$

where V is the normalization volume.

Since $S_{tot}^2 = (S_1 + S_2)^2 = 4 + 2S_1 S_2$, the average value of $S_1 S_2$ is equal to

$$S_1 S_2 = S_{tot}^2 / 2 - 2. \quad (31)$$

For $S_{tot} = 2$ this term is $S_1 S_2 = 1 > 0$, while for $S_{tot} = 0$ it is $S_1 S_2 = -2 < 0$. Thus, if the spin-spin interaction is dominated by the interactions between the magnetic moments of W bosons, the state with their maximum total spin is more favorable energetically and W -bosons should condense in the ferromagnetic state. This could lead to the spontaneous magnetization in the early universe.

4.2 Quartic self-coupling of W

The contribution to the spin-spin interactions of W comes from the first term in Lagrangian (16) or from the third term in the r.h.s. of equation of motion (18). The first term in Lagrangian (16) can be rewritten as:

$$L_{4W} = -\frac{e^2}{2\sin^2 \theta_W} \left[(W_\mu^\dagger W^\mu)^2 - W_\mu^\dagger W^{\mu\dagger} W_\nu W^\nu \right] = \frac{e^2}{2\sin^2 \theta_W} (\mathbf{W}^\dagger \times \mathbf{W})^2. \quad (32)$$

It is assumed here that $\partial_\mu W^\mu = 0$ and thus only the spatial 3-vector \mathbf{W} is non-vanishing, while $W_t = 0$.

Since the corresponding Hamiltonian is obtained from L_{4W} by changing sign and since spin operator (27) contains imaginary unit factor, the sign of the Hamiltonian is positive. It means that the low spin states are energetically favorable.

For the comparison of this Hamiltonian with potential energy (29) we need to properly normalize the wave functions of W . In the Hamiltonian the usual relativistic normalization is used, according to which the number density of W is equal to $n_W = 2m_W \mathbf{W}^\dagger \mathbf{W}$, while in the non-relativistic Schroedinger equation the wave function is normalized to unity,

$$\int d^3 r |\psi|^2 = 1 \quad (33)$$

Accordingly the Hamiltonian should be divided by $4m_W^2$ and its Fourier transform producing the spin-spin interaction potential would be:

$$U_{4W}^{(spin)} = \frac{e^2}{8m_W^2 \sin^2 \theta_W} (\mathbf{S}_1 \mathbf{S}_2) \delta^{(3)}(\mathbf{r}). \quad (34)$$

Thus the quartic self-coupling of W contributes only to the spin-spin interaction whose sign is antiferromagnetic.

The same result can be obtained from equation of motion (18) if one takes into account that in the non-relativistic limit:

$$\partial_t^2 \mathbf{W} = (-E^2 + m_W^2) \mathbf{W} \approx 2m_W \epsilon \mathbf{W}, \quad (35)$$

where $\epsilon = (E - m_W)$ is the non-relativistic energy.

4.3 Z -boson exchange

The contribution to the spin-spin potential between a pair of W from the Z -boson exchange can be found from eq. (18) where we substitute the expression for Z_ν taken from eq. (19) in the limit of vanishing four-momentum of Z . Indeed the transferred momentum is much smaller than m_Z , and so the diagram with Z -exchange is effectively local with Z -boson propagator equal to $1/m_Z^2$. Hence the contribution from the Z -exchange in the e^2 order to eq. (18) is:

$$\partial_\mu W_i^\mu + m_W^2 W_i + 4e^2 \cot^2 \theta_W (m_W/m_Z)^2 (\mathbf{W}^\dagger \mathbf{W}) W_i + \dots = 0, \quad (36)$$

where $j = 1, 2, 3$ is the spatial vector index.

We see that the Z -boson exchange does not contribute to the spin-spin interactions of W . However, it should be kept in mind that this result is true only for the non-relativistic Z -bosons, while above the phase transition the contributions of Z bosons and photons are similar.

4.4 Plasma screening

In plasma the time-time component of the photon propagator is modified as $1/\mathbf{q}^2 \rightarrow 1/[\mathbf{q}^2 + \Pi_{00}(\mathbf{q})]$, where $\Pi_{00}(\mathbf{q})$ is the photon polarization operator in plasma. Usually $\Pi_{00} = m_D^2$, where m_D is the Debye screening mass, which is independent on \mathbf{q} . So the pole at $q^2 = 0$ shifts to an imaginary q leading to the well known effects of the Debye screening. As it was found recently [3] - [7], the presence of the charged Bose condensate drastically changes the polarization operator leading to an explicit dependence of Π_{00} on \mathbf{q} which gives rise to infrared singular terms. The modification of the propagator takes place already in the lowest order in the electromagnetic coupling, e^2 , i.e. in the one loop approximation.

On the other hand, the space-space component of the propagator remains massless, $\Pi_{ij} \sim 1/\mathbf{q}^2$. It is known to be true in pure Abelian electrodynamics in any order of perturbation theory, while in non-Abelian theories the screening may occur in higher orders of perturbation theory due to the infrared singularities [13]. The screening may potentially change the relative strength of the electromagnetic spin-spin coupling, which is affected by screening effects, with respect to local W^4 coupling which is not screened. However, in the broken phase the system is reduced to the usual electrodynamics, where screening is absent and W -bosons would condense in the ferromagnetic state. In the unbroken phase of the electroweak theory the answer is not yet known. Perturbative calculations are impossible because of the violent infrared singularities. Maybe lattice calculations analogous to those done in QCD will help.

The potential describing the magnetic spin-spin interaction is related to amplitude (28) with a modified photon propagator. Thus it can be written as:

$$U_{em}^{(spin)}(\mathbf{r}) = -\frac{e^2 \rho^2}{m_W^2} \int \frac{d^3 q}{(2\pi)^3} \frac{\exp(i\mathbf{q}\mathbf{r})}{(q^2 + \Pi_{ss}(\mathbf{q}))} \left[q^2 (\mathbf{S}_1 \cdot \mathbf{S}_2) - (\mathbf{S}_1 \cdot \mathbf{q})(\mathbf{S}_2 \cdot \mathbf{q}) \right], \quad (37)$$

where Π_{ss} is the plasma correction to the space-space component of the photon propagator.

If, as above, we assume that the wave function of W -bosons is space independent and average this potential over space, we obtain the following expression for the spin-spin part of the energy shift:

$$\delta E = \int \frac{d^3 r}{V} U_{em}^{(spin)}(\mathbf{r}) = -\frac{e^2 \rho^2}{V m_W^2} \int \frac{d^3 q}{(2\pi)^3} \delta^{(3)}(\mathbf{q}) \frac{q^2 (\mathbf{S}_1 \cdot \mathbf{S}_2) - (\mathbf{q} \cdot \mathbf{S}_1)(\mathbf{q} \cdot \mathbf{S}_2)}{q^2 + \Pi_{ss}(\mathbf{q})} \quad (38)$$

Clearly δE vanishes if Π_{ss} is non-zero at $q = 0$. Of course, this is an unphysical conclusion, because the integration over r should be done with some finite upper limit, $r_{max} = l$, presumably equal to the average distance between the W bosons. So instead of the delta-function, $\delta^{(3)}(\mathbf{q})$, we obtain:

$$\int_0^l d^3 r \exp(i\mathbf{q}\mathbf{r}) = \frac{4\pi}{q^3} [\sin(ql) - ql \cos(ql)]. \quad (39)$$

The energy shift is given by the expression:

$$\delta E = -4\pi \frac{e^2 \rho^2}{V m_W^2} S_1^i S_2^j \int \frac{d^3 q}{(2\pi)^3} \frac{[\sin(ql) - ql \cos(ql)] [q^2 \delta_{ij} - q^i q^j]}{q^3 [q^2 + \Pi_{ss}(q)]}, \quad (40)$$

where $V = 4\pi l^3/3$.

When we average over an angle independent wave function, e.g. S-wave for the condensate, the non-vanishing part of the integral in Eq. (40) is proportional to the Kronecker delta, hence:

$$\delta E = S_1^i S_2^j A \delta^{ij}, \quad (41)$$

where the coefficient A can be calculated by taking trace of Eq. (40):

$$Tr(A\delta^{ij}) = 3A = -8\pi \frac{e^2 \rho^2}{m_W^2} \int \frac{d^3 q}{(2\pi)^3} \frac{[q \sin(ql) - q^2 l \cos(ql)]}{q^2 [q^2 + \Pi_{ss}(q)]}. \quad (42)$$

Hence the energy shift of a pair of W -bosons in S-wave state due to the spin-spin interaction is:

$$\delta E = -(\mathbf{S}_1 \cdot \mathbf{S}_2) \frac{8\pi e^2 \rho^2}{3V m_W^2} \int \frac{d^3 q}{(2\pi)^3} \frac{[\sin(ql) - ql \cos(ql)]}{q [q^2 + \Pi_{ss}(q)]}, \quad (43)$$

Introducing the new integration variable $x = ql$ we can rewrite it as:

$$\delta E = -(\mathbf{S}_1 \cdot \mathbf{S}_2) \frac{4e^2 \rho^2}{3\pi V m_W^2} \int_0^\infty \frac{dx}{x^2 + l^2 \Pi_{ss}(x/l)} [x \sin x + l^2 \Pi_{ss}(x/l) \cos x], \quad (44)$$

We used here the usual regularization of divergent integrals: $\exp(\pm iql) \rightarrow \exp(\pm iql - \epsilon q)$ with $\epsilon \rightarrow 0$. With such regularization $\int_0^\infty dx \cos(x) = 0$.

Evidently, if $\Pi_{ss} = 0$, we obtain the same result as that found in sec. 4.1. In fact the necessary condition for obtaining the “unscreened” result is $l^2 \Pi_{ss}(x/l) \ll 1$, but for a large $l^2 \Pi_{ss}$ the electromagnetic part of the spin-spin interaction can be suppressed enough to change the ferromagnetic behaviour into the antiferromagnetic one. This might take place at high temperatures above the EW phase transition when the Higgs condensate is destroyed and the masses of W and Z appear only as a result of temperature and density corrections and thus are relatively small. The quantitative statement depends upon the modification of the space-space part of the photon propagator in presence of the Bose condensate of charged W . As far as we know, this modification has not yet been found.

4.5 Discussion

It is instructive to compare the magnetic properties of solid state bodies with the considered here (anti-)ferromagnetism of BEC of W -bosons. Magnetic properties of matter are determined by the state of outer (unpaired) electrons. A pair of electrons belonging to different atoms may be either in symmetric, $S_{tot} = 1$, or antisymmetric, $S_{tot} = 0$, spin state. Since the total wave function of two electrons must be antisymmetric, their spin state has opposite symmetry with respect to their orbital wave function. Symmetric and antisymmetric electron states evidently have different energies, which we denote as E_s and E_a respectively. Accordingly the spin Hamiltonian can be written as

$$H^{spin} = -J \mathbf{S}_1 \cdot \mathbf{S}_2. \quad (45)$$

The quantity $J = (E_s - E_a)$ is usually called the exchange energy. Its sign is determined by the atomic ground state structure. Evidently $J > 0$ favors parallel spins, while $J < 0$ favors antiparallel spins. In atomic systems the exchange energy at small distances is typically of the order of fractions of eV, that is about 10^3 times larger than the typical direct magnetic dipole (spin-spin) interaction between electrons. Hence the exchange interaction may force the magnetic dipoles of electrons to be aligned or anti-aligned independently on their direct magnetic interaction. The situation changes at large distances, since the exchange energy decays exponentially, while the magnetic interaction behaves like r^{-3} . Thus the latter dominates on macroscopic scales, leading to the breaking of the system into domains with different directions of the magnetic field and consequently to zero net macroscopic magnetization. Nevertheless, the ferromagnetic nature emerges when an external magnetic field is applied to the system.

Fortunately, the system we are considering here is much simpler. All W -bosons are in the same state with zero momentum and are not binded into a complicated atomic system. Evidently the exchange forces are not essential in this case. Indeed, W -bosons are in symmetric orbital state. Hence their spin wave function should also be symmetric and both the allowed spin states of W , the scalar, $S = 0$, and the tensor, $S = 2$, ones are also symmetric.

In principle, electrons and positrons could distort the spin-spin interactions of W by their spin or orbital motion and thus destroy the attraction of parallel spins of W . However, it looks hardly possible because electrons are predominantly ultra-relativistic and they cannot be attached to any single W boson to counterweight its spin. The low energy electrons cannot be long in such a state because of fast energy exchange with the energetic electrons. The scattering of electrons (and quarks) on W -bosons may lead to the spin flip of the latter, but in thermal equilibrium this process does not change the average value of the spin of the condensate.

5 Conclusion

We have studied here the Bose-Einstein condensation of charged weak bosons, W^\pm . Such condensation might occur in the early universe if the cosmological lepton asymmetry was sufficiently large. The primeval plasma is supposed to be electrically neutral and to have zero density of weak hypercharge.

In general a Bose-Einstein condensate of vector fields may form either a scalar state, when the average value of vector W_j is microscopically small, or a classical vector state when all the individual spins of the condensed vector bosons are aligned at a macroscopically large scale.

In solid state physics the realization of one or the other of these two possibilities depends on the short range exchange forces which dominate over the direct spin-spin interaction at small

distances. In the case of W bosons the choice between ferromagnetic or antiferromagnetic state is determined by the spin-spin interaction of the individual W -bosons, realized through the interactions of their magnetic moments and their quartic self-coupling.

The total spin-spin interaction potential for W is the sum of two terms (29) and (34). As we have discussed below eq. (29), the first two terms in the interaction of the magnetic moments cancel each other after averaging over a S-wave function. Thus only the δ -function term survives. In the standard electroweak model $\rho = 1$ and thus the absolute value of the coefficient in front of $\mathbf{S}_1\mathbf{S}_2\delta^{(3)}(\mathbf{r})$ in eq. (29) is $2e^2/(3m_W^2)$ which is larger than the corresponding term in eq. (34). Hence the former dominates and the energetically favored configuration of a multi- W state should have a macroscopically large total spin. However, as we have pointed out in Sec.4.4, the plasma screening of the interaction between the magnetic moments may be dangerous for the W -ferromagnetism. In the broken phase the problem is reduced to that of pure QED, where it is known that magnetic forces are not screened. However, in non-Abelian gauge theories the absence of screening is known only in the lowest order in perturbation theory, while higher order calculations suffer from strong infrared divergences and are not reliable. For the resolution of this problem non-perturbative methods, as e.g. lattice calculations, are necessary. At the moment the problem remains unsolved.

It should be also noted that, even though Eqs. (23) and (29) are calculated for electrons and W bosons, they are valid respectively for any spin-(1/2) and spin-1 species having the usual electromagnetic interactions. So they may be applied to other particles, including the ones present in the extensions of the standard model. If these particles are in a S-wave state, on the average the only delta term survives and hence they may form ferromagnetic states. Indeed, as one can see from Eq. (31), the lowest energy state for a S-wave function is the one with the maximum S_{tot}^2 .

It is clear from Eqs. (29) and (34) that the condensate of W -bosons would be antiferromagnetic if the bosons have a negative non-standard contribution into the magnetic moment, such that $\rho < 3/(16\sin^2\theta_W)$. The ferromagnetism of W can be destroyed also in a model with a smaller value of the Weinberg angle. All that demands a strong deviation from the standard model and most probably is excluded, but these effects may be important in applications to extensions of the standard model, e.g. SUSY.

If a ferromagnetic state is formed, we would expect that the primeval plasma, where such bosons condensed (maybe due to a large cosmological lepton asymmetry), can be spontaneously magnetized. The typical size of the magnetic domains is determined by the cosmological horizon at the moment of the condensate evaporation. The latter takes place when the neutrino chemical potential, which scales as temperature in the course of cosmological cooling down, becomes smaller than the W mass at this temperature. However, if during the electroweak phase transition a very strong chaotic magnetic field was generated, the alignment of the spins of W -bosons and the domain size would be determined by this magnetic field. In the course of the cosmological expansion such field would drop down as the cosmological scale factor squared and the spins of W -bosons would behave as considered above. That is, their dynamics would be determined by the spin-spin interactions and microscopically small magnetic domains would rearrange themselves into macroscopically large ones in the same way as it happens in the usual ferromagnets.

Large scale magnetic field created by the ferromagnetism of W -bosons might survive after the decay of the condensate due to the conservation of the magnetic flux in plasma with high electric conductivity. Such magnetic fields at macroscopically large scales may be the seeds of the observed larger scale galactic or intergalactic magnetic fields. This problem will be studied elsewhere, we only note here that the mechanism of generation of galactic or intergalactic magnetic fields is

unknown and presents a long standing cosmological problem, for reviews see e.g. ref. [35]. Seed magnetic fields generated during inflation could be quite easily uniform at galactic or intergalactic scales, but they are too weak, hence a huge galactic dynamo is necessary to amplify them up to the observed magnitude. However, it is impossible in this way to explain the existence of intergalactic fields. On the other hand, seed fields generated at later stages of cosmological evolution could be quite large (e.g. magnetic fields created at some cosmological phase transitions), but the characteristic scale of such fields is by far smaller than the galactic one. The mechanism suggested here may generate large magnetic fields and at scales which are of the order of cosmological horizon at the electroweak temperatures. In this sense the mechanism is unique. Still even after the cosmological stretching up of the characteristic wave length of the field, it remains smaller than the galactic radius. Nonetheless, with the “Brownian motion” reconnection of the field lines, the characteristic scale can be enlarged up to the galactic scale, though by an expense of their magnitude. Nevertheless with rather mild galactic dynamo the observed magnetic field may be generated.

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